Oxford Cambridge and RSA

Wednesday 21 October 2020 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

8314192568

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

1 Fig. 1 shows some spreadsheet output.

	A
1	1E-17
2	1E-17
3	1E-29

Fig. 1

(a) Write the value displayed in cell A3 in standard mathematical notation. **[1]**

The formula in cell A3 is

= A2 − A1

(b) Explain why the value displayed in cell A3 is non zero. **[1]**

(c) Write down the value of the number stored in cell A2 to the highest precision possible. **[1]**

(d) Explain why your answer to part **(c)** may be different to the actual value stored in cell A2. **[1]**

2 Fig. 2 shows 3 values of x and the associated values of a function, $\mathrm{f}(x)$.

x	1	2	5
$\mathrm{f}(x)$	5	16.6	76.6

Fig. 2

Find a polynomial $p(x)$ of degree 2 to approximate $\mathrm{f}(x)$, giving your answer in the form
$p(x) = ax^2 + bx + c$,
where a, b and c are constants to be determined. **[5]**

3 At Heathwick airport each passenger's luggage is weighed before being loaded into the hold of the aeroplane. Each weight is displayed digitally in kg to 1 decimal place. Some examples are given in Fig. 3.

Weight (kg)
17.2
19.9
22.3
20.1
21.5

Fig. 3

On each flight, the total weight of luggage is calculated to ensure compliance with health and safety regulations.

Winston models this situation by assuming that the displayed weights are **rounded** to 1 decimal place, and that the **total** weight of luggage is calculated using the displayed values.

On a flight to Athens, there are 154 items of passengers' luggage.

(a) Determine the maximum possible error, according to Winston's model, when the **total** weight of luggage is calculated for the flight to Athens. **[2]**

Piotre models this situation by assuming that the displayed weights are **chopped** to 1 decimal place, and that the **total** weight of luggage is calculated using the displayed values.

(b) Determine the maximum possible error, according to Piotre's model, when the total weight of luggage is calculated for the flight to Athens. **[2]**

A health and safety inspector notes that the total of the displayed weights is 3080.2 kg. However, when the luggage is all weighed together in the loading bay, the total weight is found to be 3089.44 kg.

(c) Determine whether Winston's model or Piotre's model is a better fit for the data. **[3]**

4 **(a)** Use the trapezium rule with 1 strip to calculate an estimate of $\int_1^2 \sqrt{1+x^3}\,dx$, giving your answer correct to six decimal places. **[2]**

Fig. 4 shows some spreadsheet output containing further approximations to this integral using the trapezium rule, denoted by T_n, and Simpson's rule, denoted by S_{2n}.

	A	B	C
1	n	T_n	S_{2n}
2	1		2.130135
3	2	2.149378	
4	4	2.134751	2.129862
5	8	2.131084	

Fig. 4

(b) Write down an efficient formula for cell C4. **[2]**

(c) Find the value of S_4, giving your answer correct to 6 decimal places. **[2]**

(d) **Without** doing any further calculation, state the value of $\int_1^2 \sqrt{1+x^3}\,dx$ as accurately as possible, justifying the precision quoted. **[2]**

(e) Use the fact that Simpson's rule is a fourth order method to obtain an improved approximation to the value of $\int_1^2 \sqrt{1+x^3}\,dx$, stating the value of this integral to a precision which seems justified. **[2]**

5 You are given that

$$g(x) = \frac{\sqrt[3]{x^x + 25}}{2}.$$

Fig. 5.1 shows two values of x and the associated values of $g(x)$.

x	1.45	1.55
$g(x)$	1.49468	1.49949

Fig. 5.1

(a) Use the central difference method to calculate an estimate of $g'(1.5)$, giving your answer correct to 3 decimal places. **[2]**

The equation $x^x - 8x^3 + 25 = 0$ has two roots, α and β, such that $\alpha \approx 1.5$ and $\beta \approx 4.4$.

(b) Obtain the iterative formula $x_{n+1} = g(x_n) = \dfrac{\sqrt[3]{x_n^{x_n} + 25}}{2}$. **[2]**

(c) Use your answer to part **(a)** to explain why it is possible that the iterative formula $x_{n+1} = g(x_n) = \dfrac{\sqrt[3]{x_n^{x_n} + 25}}{2}$ may be used to find α. **[1]**

(d) Starting with $x_0 = 1.5$, use the iterative formula to find x_1, x_2, x_3, x_4, x_5, and x_6. **[2]**

(e) Use your answer to part **(d)** to state the value of α correct to 8 decimal places. **[1]**

Starting with $x_0 = 4.5$ the same iterative formula is used in an attempt to find β. The results are shown in Fig. 5.2.

n	x_n
0	4.5
1	4.81826433
2	6.27473453
3	23.2937196
4	2.0654E+10
5	#NUM!

Fig. 5.2

(f) Explain why #NUM! is displayed in the cell for x_5. **[1]**

(g) On the diagram in the Printed Answer Booklet, starting with $x_0 = 4.5$, illustrate how the iterative formula works to find x_1 and x_2. **[1]**

(h) Determine what happens when the relaxed iteration $x_{n+1} = (1-\lambda)x_n + \lambda g(x_n)$ is used to try to find β with $x_0 = 4.5$, in each of the following cases.

- $\lambda = 0.5$
- $\lambda = -0.4$ **[3]**

6 Fig. 6.1 shows the graph of $y = e^{3x} - 11x - 0.5$ for $-0.5 \leqslant x \leqslant 1$.

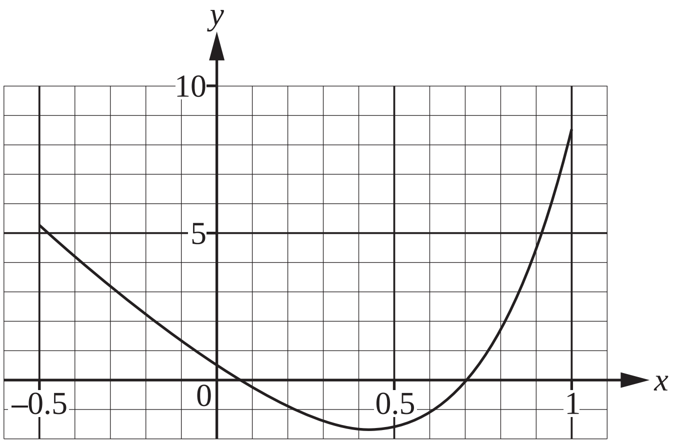

Fig. 6.1

The equation $e^{3x} - 11x - 0.5 = 0$ has two roots, α and β, such that $\alpha < \beta$. Dennis is going to use the method of interval bisection with starting values denoted by a and b.

(a) Explain why the method of interval bisection starting with $a = 0$ and $b = 1$ may not be used to find either α or β. **[1]**

Dennis uses the method of interval bisection starting with $a = 0$ and $b = 0.5$ to find α. Some spreadsheet output is shown in Fig. 6.2.

	A	B	C	D	E	F
1	a	f(a)	b	f(b)	x_{new}	f(x_{new})
2	0	0.5	0.5	–1.51831	0.25	–1.133
3	0	0.5	0.25	–1.133	0.125	–0.42
4	0	0.5	0.125	–0.42001	0.0625	0.01873
5	0.0625	0.01873	0.125	–0.42001	0.09375	–0.2065

Fig. 6.2

Dennis states that the formula in cell B2 is

= EXP(3 * A1) – 11A2 – 0.5

Dennis has made two errors.

(b) Write a correct version of Dennis's formula for cell B2. **[1]**

The formula in cell A3, which is correct, is

= IF(F2 > 0, E2, A2)

(c) Write a suitable formula for cell C3. **[1]**

(d) Use the information in Fig. 6.2 to

- find the value of α as accurately as possible,
- state the maximum possible error in this estimate. **[2]**

Liren uses a different method to find a sequence of estimates of the value of β using a spreadsheet. The output, together with some further analysis, is shown in Fig. 6.3.

	A	B	C	D
1	x	$f(x)$	difference	ratio
2	0.4	–1.5799		
3	0.6	–1.0504		
4	0.99671	8.4245		
5	0.64398	–0.6809	–0.35273	
6	0.67036	–0.4026	0.026378	–0.0748
7	0.70852	0.08386	0.038164	1.44682
8	0.70194	–0.0075	–0.00658	–0.1724
9	0.70248	–0.0001	0.00054	–0.082
10	0.70249	1.8E-07	8.88E-06	0.01646

Fig. 6.3

The formula in cell A4 is

= (A2 * B3 – A3 * B2)/(B3 – B2)

(e) State the method being used. **[1]**

(f) Explain what the values in column D tell you about the order of convergence of this sequence of estimates. **[2]**

Liren states that $\beta = 0.70249$ correct to 5 decimal places.

(g) Determine whether Liren is correct. **[2]**

7 Fig. 7.1 shows two values of x and the associated values of $f(x)$.

x	3	3.5
$f(x)$	6.082763	4.596194

Fig. 7.1

(a) Use the forward difference method to calculate an estimate of the gradient of $f(x)$ at $x = 3$, giving your answer correct to 4 decimal places. **[2]**

Fig. 7.2 shows some spreadsheet output with additional values of x and the associated values of $f(x)$.

x	3	3.00001	3.0001	3.001	3.01	3.1
$f(x)$	6.082763	6.08274	6.082541	6.08054	6.060454	5.848846

Fig. 7.2

These values have been used to produce a sequence of estimates of the gradient of $f(x)$ at $x = 3$, together with some further analysis. This is shown in the spreadsheet output in Fig. 7.3.

h	0.1	0.01	0.001	0.0001	0.00001
estimate	–2.339165	–2.230883	–2.220532	–2.219501	–2.219398
difference	0.1082815	0.010352	0.0010307	0.000103	
ratio	0.095602	0.099567	0.0999568		

Fig. 7.3

Tommy states that the differences between successive estimates is decreasing so rapidly that the order of convergence of this sequence of estimates is much faster than first order.

(b) Explain whether or not Tommy is correct. **[2]**

(c) Use extrapolation to determine the value of the gradient of $f(x)$ at $x = 3$ as accurately as possible, justifying the precision quoted. **[5]**

(d) Calculate an estimate of the absolute error when $f(3)$ is used as an approximation to $f(3.02)$. **[2]**

END OF QUESTION PAPER

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